1. Beweise

   a)

   $$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

   b)

   $$|z_1 + z_2| \leq |z_1| + |z_2|$$

   c)

   $$||z_1| - |z_2|| \leq |z_1 - z_2|$$

   d)

   $$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \quad z_2 \neq 0$$

2. Beweise

   a)

   $$\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$$

   b)

   $$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

3. Bestimme z.

   a)

   $$|z| = |z + 5|, \ \Im(z) = 2$$

   b)

   $$|z| = |z + 4i|, \ \Re(z) = 3$$

   c)

   $$|z| = |2z - 3|, \ \Im(z) = -1$$

d)

$$|z| = |2z - 3|,\ \Re(z) = 2$$

4. Skizziere, welche Teilmenge der GAUSSschen Zahlenebene beschrieben wird! Dabei ist zu beachten, daß $0 \leq arg(z) < 2\pi$.

a)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z| < 2\}$$

b)

$$\mathcal{M} = \{z \in \mathbb{C} \mid arg(z) < \pi\}$$

c)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z| < 0\}$$

d)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z| > 1 \wedge arg(z) < \frac{\pi}{2}\}$$

e)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z| \geq 3\}$$

f)

$$\mathcal{M} = \{z \in \mathbb{C} \mid arg(z) = -\pi\}$$

g)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z| \leq 3 \wedge arg(z) > \pi\}$$

h)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z - 1 + i| \leq 1\}$$

i)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z + 1 - 2i| = 5\}$$

j)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z - 3| = |z - i|\}$$

k)

$$\mathcal{M} = \{z \in \mathbb{C} \mid 0 < |z| \leq 2\}$$

l)

$$\mathcal{M} = \{z \in \mathbb{C} \mid 1 \leq |z - 1 + 2i| \leq 2\}$$

m)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z + 1 - 2i| \leq 1\}$$

n)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z - (-1 + 2i)| \leq 2\}$$

o)

$$\mathcal{M} = \{z \in \mathbb{C} \mid 1 \leq |z + 1 - 2i| \leq 2\}$$

p)

$$\mathcal{M} = \{z \in \mathbb{C} \mid |z + 1 - 2i| \leq 2\}$$