

## Approximation von Funktionen

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**Satz von Taylor:** Es sei  $I$  ein beliebiges Intervall und  $x_0 \in I$  im Innern von  $I$ .  $f$  sei  $(n + 1)$ -mal differenzierbar auf  $I$ . Dann gilt:

$$f(x) = \sum_{i=0}^n \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i + R_n(x, x_0) \quad \text{mit} \quad R_n(x, x_0) = \frac{1}{n!} \int_{x_0}^x (x - t)^n f^{(n+1)}(t) dt$$

TAYLOR-Entwicklungen einiger wichtiger Funktionen:

$$\begin{aligned}
 e^x &= 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots &= \sum_{n=0}^{\infty} \frac{x^n}{n!} & ; \quad x \in \mathbb{R} \\
 \sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} & ; \quad x \in \mathbb{R} \\
 \cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \dots &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} & ; \quad x \in \mathbb{R} \\
 \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots &= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} & ; \quad -1 < x \leq 1 \\
 \ln(1-x) &= -(x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \dots) &= -\sum_{n=1}^{\infty} \frac{x^n}{n} & ; \quad -1 \leq x < 1 \\
 \ln\left(\frac{1+x}{1-x}\right) &= 2\left(x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \frac{1}{7}x^7 + \dots\right) &= 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} & ; \quad -1 < x < 1 \\
 \sinh x &= x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7 + \dots &= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} & ; \quad x \in \mathbb{R} \\
 \cosh x &= 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \dots &= \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} & ; \quad x \in \mathbb{R} \\
 \arctan x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} & ; \quad -1 \leq x \leq 1 \\
 (1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{1 \cdot 2} x^2 \\
 &\quad + \frac{\alpha(\alpha-1)(\alpha-2)}{1 \cdot 2 \cdot 3} x^3 + \dots & & ; \quad \alpha \neq 0 \\
 &= \binom{\alpha}{0} + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \binom{\alpha}{3}x^3 + \dots = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n & ; \quad -1 < x < 1
 \end{aligned}$$